VISCOPLASTIC CONSTITUTIVE EQUATIONS FOR COPPER WITH STRAIN RATE HISTORY AND TEMPERATURE EFFECTS

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S. R. BODNER AND A. MERZER

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by

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Faculty of Mechanical Engineering
Technion - Israel Institute of Technology

ABSTRACT

Elastic-viscoplastic constitutive equations based on a single internal state variable which is a function of plastic work are used to calculate the response of copper to a six decade change of strain rates over a range of temperatures. Calculations were performed for the conditions of an experimental program on copper by Senseny, Duffy, and Hawley, namely, temperatures ranging from 77 to 523°K and strain rate jumps from $2 \times 10^{-4} sec^{-1}$ to $3 \times 10^{2} sec^{-1}$ at three strain levels. The computed results are in good agreement with the experiments and show similar strain rate and strain rate history effects. Relations are obtained for the temperature dependence of certain parameters in the equations which indicate correspondence between plastic working and temperature and between strain rate sensitivity and temperature.

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1 Professor

2 Senior Lecturer

INTRODUCTION

Recent formulations of elastic-viscoplastic constitutive equations, e.g. [1,2,3,4] propose to represent rate dependent material response for a wide range of loading histories. A common approach in most of these theories is that all aspects of non-reversible deformation are lumped into a single equation relating the inelastic deformation rate to stress, temperature, and one or more internal state variables. When this is taken together with the elastic deformation rate equation, the total deformation rate can be expressed as a function of state variables and the current geometry.

Most of the recent work on viscoplastic constitutive equations has been motivated by the need to develop suitable creep equations under conditions of fluctuating load, strain, or temperature. The form of the equations and the manner of including temperature dependence are therefore similar to high temperature creep equations. Other formulations, in particular that of Bodner and Partom [1,5,6], were initially directed towards representing strain rate effects over a wide range of conditions. Those equations serve to calculate stress-strain curves for imposed constant and varying strain rates. Although not specifically intended for creep, the equations can also represent creep and stress relaxation behavior [1].

The intention of the investigation described in this paper was to examine the capability of the constitutive equations of Bodner and Partom to represent a relatively rate insensitive metal, copper, over a wide range of strain rates and temperatures. These equations are based on a single parameter to characterize the microstructural state with respect to inelastic deformation. A recent experimental program by Senseny, Duffy, and Hawley [7] provides shear stress-strain curves for annealed (640°K) OFHC copper under static ($2 \times 10^{-4} sec^{-1}$) and dynamic ($3 \times 10^{2} sec^{-1}$) conditions for temperatures ranging from 77°K to 523°K. Tests were also conducted in [7] for rapid changes from the low to the high strain rate. Similar experiments have also been conducted by Eleiche and Campbell [8] using differently prepared copper specimens. A number of other investigators have obtained static and dynamic stress-strain curves for OFHC copper with comparable results. For consistency, it was decided to use a single set of results, namely [7], as the reference for this study.

The steady strain rate curves of [7] were used as the basis for determining the material constants in the constitutive equations at the various temperatures. With these constants, response curves for rapid changes of strain rate were calculated. Their relationship to corresponding test results was taken as a measure of the applicability of the equations. The equations have also been used to predict creep response under constant and varying load conditions and this will be reported in a subsequent paper. For this case, a recovery term was added to the basic equations.

The empirical determination of the material constants led to relatively simple analytical forms for their temperature variation. These should be useful for calculations involving varying temperature conditions and in identifying the operative microscopic deformation mechanisms.

DETERMINATION OF MATERIAL CONSTANTS

Copper is relatively insensitive to rate of strain effects and exhibits fairly strong strain hardening. The experiments reported in [7] cover six decades of strain rate and show strain rate and strain rate history effects which are emphasized in the tests involving rapid changes of strain rates. Rather than hypothesizing the manner of including temperature dependence in the constitutive equations, an empirical approach was followed. That is, numerical exercises were undertaken to find suitable values for the material constants so that the equations would represent the experimental results. An objective in these exercises was to minimize the number of constants that were temperature dependent.

The basic equations in the formulation of Bodner and Partom [1] relate the reversible and non-reversible components of the deformation rate, which are taken to be always non-zero, to state quantities. For small strains, the reversible, i.e. elastic, deformation rate is given by the time derivative of Hooke's Law. Since the tests used for reference were in pure shear, only the shear modulus G is required. The temperature variation of G is small over the range of the tests and was not significant in the numerical calculations. At room temperature, G for copper is 45000 MPa.

A general form for the non-reversible, i.e. inelastic, deformation rate, which is equal to the strain rate for small strains, is,

$$d^p_{ij} = f(\sigma^e_{ij}, T, Z_k) \tag{1}$$

where σ^e_{ij} are the elastic stress components, T is the temperature, and Z_k are one or more internal state variables. From the flow law of classical isotropic plasticity,

$$d^p_{ij} = \lambda s_{ij} \tag{2}$$

it follows, upon squaring, that

$$D^p_2 = \lambda^2 J_2 \tag{3}$$

where D^p_2 is the second invariant of the plastic deformation rate and J_2 is the second invariant of the stress deviator. A fundamental postulate in the formulation of [1,5,6] is that plastic deformation is governed by a continuous relation between D^p_2 and J_2 involving T and Z_k. Solving for λ in (3) and substituting into (2) then gives an equation for d^p_{ij} in the form of (1)

$$d^p_{ij} = [F_1(J_2, T, Z_k)/J_2]^{1/2} s_{ij} \tag{4}$$

The expression chosen for F_1 was motivated by equations proposed to relate dislocation velocity to stress, namely,

$$D^p_2 = F_1(J_2, T, Z_k) = D_0^2 \exp[-(\tfrac{1}{3}Z^2)^n (\tfrac{n+1}{n})/J^n_2] \tag{5}$$

where some of the constants are temperature dependent. The parameter Z in Eq. (5) can be interpreted as an internal state variable which provides a measure of the overall resistance to plastic flow. It has the dimension of stress and can be referred to as the "hardness". As such, it is similar to the hardness state variable of Hart [2] but is treated in a different manner. Eq. (5) is shown in Fig. 1 for various Z values.

In Eq. (5), D_0 is a scale factor and n relates to strain rate sensitivity with higher n meaning lower sensitivity. Additional constants arise in the evolution equation for Z which was taken to be a function of plastic work W_p. In representing titanium for various uniaxial straining histories [1], the function

$$Z = Z_1 + (Z_0 - Z_1)\exp(-mW_p) \tag{6}$$

which introduces three more constants, was adequate. This was not the case for copper which exhibits strong strain hardening, so m was made a function of W_p,

$$m = m_0 + m_1\exp(-\alpha W_p) \tag{7}$$

which adds two more constants. It is likely that fewer than five constants are actually needed for $Z(W_p)$ for copper, Fig. 2, by other choices of the basic functional form (6), e.g. a polynomial, but this was not attempted here.

Consideration of temperature dependence in the equation for the inelastic strain rate has been studied extensively under creep conditions and suggestions on similar relationships have been proposed for steady plastic flow at moderate temperatures. High temperature ($T > .5T_m$) creep equations are generally presented in the form of (1) in which the temperature effect appears as a simple multiplying factor (1/kT) where k is Boltzmann's constant. A general form for temperature-strain rate correspondence was proposed in [9], namely that steady rate stress-strain curves depended on the single parameter $\dot{\varepsilon}\exp(\Delta H/RT)$. This correspondence was shown in [9] to be applicable for pure aluminum over a range of constant strain rates and temperatures. More recently, Campbell [10] presented a detailed discussion on temperature-strain rate relations in metal plasticity. He developed expressions for the inclusion of temperature effects in strain rate-stress equations based on models for deformation mechanisms. Some of these expressions could be expected to apply for steady rate conditions but, as Campbell also emphasizes, may not be applicable for histories involving changing strain rates.

Following the extensive work on creep and the correspondence law proposed in [9], it seemed reasonable to consider D_0 and n in (5) to be functions of temperature. Initially a number of numerical exercises were performed on this basis, but the resulting functions for $D_0(T)$ and $n(T)$ were not reasonable, e.g. D_0 varied with temperature by a few orders of magnitude. Further exercises based on varying only $Z(W_p)$ and n with temperature led to the conclusion that the function $Z(W_p)$ could be considered generally applicable with the initial state Z_i being temperature dependent. Applicability of a single $Z(W_p)$ curve implies a temperature-hardness correspondence in the constitutive equations. Lowering the operating temperature would be equivalent to increasing the initial hardness value which could have been achieved by prior work hardening. One of the results reported in [9] does imply this effect. Since Z_i is the stable hardness value at a given temperature, the recovery (annealing) equation of hardening due to plastic work would have the form

$$\dot{Z} = F_2[Z(W_p) - Z_i] \tag{8}$$

Obtaining the material constants by matching calculated and experimental stress-strain curves could be done on a trial and error basis or more formally by computations using a least squares procedure. In general, a pair of stress-strain curves at two different steady rates are necessary and sufficient to provide essentially unique values for all the material constants at a given temperature. Some variation in the values would result from different criteria of curve matching. Accuracy of matching curves was attempted to within the experimental accuracy which was given as 5% in [8] and was probably similar in [7].

Since the experimental values of the initial stress increment due to the jump in strain rate, $\Delta\tau$ (Fig. 1), should be more exact than the gap between the two steady rate curves, it would seem preferable to use the $(\Delta\tau)$'s in conjunction with the lower rate curve to obtain the material constants. However, the procedure of forcing agreement of the respective $\Delta\tau$ values tends to lead to overall distortion of the steady rate curves. A number of numerical exercises were performed using this approach but were discontinued.

The material constants obtained at the various test temperatures are listed in Table 1. As mentioned, these are not precisely unique values and limited variations of combinations of the constants lead to almost identical results. The function $Z(W_p)$ based on these constants is shown in Fig. 2 with the values of Z_i at the test temperatures indicated. Calculated steady rate stress-strain curves are shown in Figs. 3-6 with corresponding test results from [7]. Superimposed on these curves are the respective results for rapid changes of strain rate. A detailed "blow up" of one of the calculated jump curves is shown in Fig. 7.

As noted in Table 1, the material constants that were found to vary with temperature were Z_i and n. Plotting Z_i against the reciprocal of absolute temperature leads to a straight line, Fig. 8, which can be expressed analytically as

$$Z_i(\text{MPa}) = (5000 + 15T)/T(°K) \tag{9}$$

A plot of n directly against T, Fig. 9, suggests a linear relationship which can be expressed as

$$n = -\ 0.025T(°K) + 16.6 \tag{10a}$$

Although Eq. (10a) provides a good fit to the derived n values, it is physically unrealistic outside the range of test temperatures since it is expected that $n \to \infty$ as $T \to 0$ and n should be small but non-zero as $T \to T_m$. A hyperbolic or piecewise linear relationship for n(T) would be more meaningful. A least squares linear fit of n(1/T) gives

$$n = (835 + 5.0T)/T(°K) \tag{10b}$$

with fairly large deviations. These deviations appear, Fig. 9, to be greater than expected from experimental scatter and variations in curve matching criteria, but a hyperbolic relation such as (10b) may be a reasonable approximation over a large range of T.

Recovery of hardness (annealing) was not considered in calculating the stress-strain curves but would be important under creep conditions. In fact, these constitutive equations with a recovery term added would show the generally accepted result that secondary creep is a balanced condition between hardness increases due to straining and decreases due to recovery. An equation for hardness recovery suggested in the literature in the form of (8) is

$$\dot{Z} = -C(Z-Z_i)^r \tag{11}$$

with r an integer. For a particular r, C could be specified so that the equations would lead to a prescribed constant creep rate at a particular stress and temperature.

Inclusion of the recovery term (11) also enables calculation of transient effects during creep due to increments or decrements of load or temperature. An interesting phenomenon is the delay time consequent to a small load decrement, i.e., a period of essentially zero strain followed by creep at the lower stress level. Preliminary calculations indicate that the equations do show this effect.

DISCUSSION OF RESULTS

A comparison of the calculated and experimental results, Figs. 3-6 and Table 2 (with symbols defined in Fig. 1), leads to the tentative conclusion that, at least for copper, viscoplastic constitutive equations based on a single internal state variable which is a function of plastic work are adequate to represent strain rate history effects associated with rapid changes of strain rate. In Figs. 3-6 and Table 2, the deviations of the calculated from the test results for the steady rate curves are generally less than the experimental accuracy of 5%. The percentage errors in comparing respective stress increment values are greater, but the overall characteristics of the calculated curves for sudden strain rate changes are similar to those of the tests.

Preference in matching the respective steady rate curves was given to the room temperature case which influenced the constants in the $Z(W_p)$ function. The calculated steady rate at 523°K also matched fairly well, but those at the lower temperatures deviated from those of the tests by more than 5% at strains higher than about 15%. For this reason, only results to 15% strain are shown for 77°K. In retrospect, it seems that somewhat better overall matching could have been achieved with a slightly higher value for Z_1 than that given in Table 1, that is, Z_1 should have been about 250 MPa. However, the results shown match the test curves fairly well are are considered adequate to represent the potentialities of the analytical representation.

Fig. 7 provides a detailed illustration of the calculated changes in the stress and the strain rate components consequent to an instantaneous jump of six decades in the applied strain rate. The example shown in Fig. 7 is for the second jump (γ_j=12%) at 298°K. Prior to the jump, at the low strain rate $\dot{\gamma}_s$, the deformation increments are almost entirely plastic (99.2%). Since the plastic strain rate $\dot{\gamma}^p$(=d^p) is a function of stress which doesn't change instantaneously, the discontinuity in total strain rate must be accomodated by the elastic strain rate $\dot{\gamma}^e$ which becomes essentially equal to the imposed rate. This results in an elastic stress increment and a rapid increase in the stress level. The plastic strain rate correspondingly increases rapidly and becomes almost equal to the new imposed rate in about 1μs (Fig. 7). During this time interval, the elastic strain rate decreases to less than 1% of the total. At all the test temperatures, the calculated readjustment in stress and strain rates was completed in less than 2μs which was taken as the jump time interval for obtaining $\Delta\tau$. Since the actual jump times in the experiments [7] were about 10μs, the initial sharp increment in stress would be slightly reduced and would be followed by steady plastic flow. Plastic work during the strain rate jump would be negligible so the parameter that represents the microstructural state, Z, would remain constant during the jump. The magnitude of the stress increment could then be calculated directly from (5) for the appropriate value of Z. In practice, $\Delta\tau$ was obtained from the calculated results based on 2μs jump time.

It is noted that if the strain rate jump were applied locally to one end of a bar, then an elastic wave would propagate due to the elastic deformation increment. This is consistent with the known result that a stress increment suddenly applied to a plastically deforming rod would propagate at the elastic bar velocity.

It is also of interest to follow the details of the strain rate jump on the D_2^p, J_2 plane, Fig. 1. With increasing plastic deformation at the lower strain rate, $\dot{\gamma}_s$, the hardness, Z, increases so that the state curve shifts on the D_2^p, J_2 plane (point A moving to B corresponding to Z' changing to Z"). At B the total strain is suddenly increased to $\dot{\gamma}_d$ so the plastic deformation rate rises in a very short time, about 2μs, to point C corresponding to the increase in stress, $\Delta J_2 [=(\Delta\tau)^2]$, at the same state value Z". Continued deformation at the high strain rate generates additional plastic flow, C to D, with increasing values of Z. The steady high strain rate curve would have state points corresponding to C and D, namely C' and D', at the same stress and plastic strain rates. The extent to which the actual incremental curve CD matches the steady curve C'D' can be viewed as a measure of the adequacy of a single internal state variable in the constitutive equations. Examining the details of the stress-strain curves for copper in [7] on this basis, the use of a single microstructural parameter seems reasonable but is not precise. However, other results from decremental temperature tests [11] tend to support the applicability of a single internal state variable. It is noted that point F in Fig. 1, which can be reached at a steady high rate, has the same strain as points B and C but corresponds to a different state.

Another measure of the validity of the representation is the comparison of the magnitude of the calculated stress increments $\Delta\tau$ with those obtained in the experiments. These are listed in Table 2. The percentage differences are greater than those between the calculated and experimental steady rate curves, but the form of dependence of the calculated stress increments with strain and temperature corresponds to those of the tests. As in the tests, the calculated stress increment increases with increasing strain, γ_j. The temperature dependence of the $\Delta\tau$ values is erratic at the lower temperatures, but the calculated results do follow the trends of the tests. Both sets of results for the stress increments are maximum at 523°K.

The calculated stress increments are higher than those of the tests by 15 to 70% based on average test values. There are a few reasons for these differences apart from using the steady rate curves as the basis for the material constants. These are: the strain rate for the dynamic incremental tests were usually lower than the nominal $\dot{\gamma}_d$=300 sec^{-1} on which the calculations were based; the 2μs time interval used for the calculated $\Delta\tau$'s included some work hardening plastic flow; the experimental time for the change of strain rate, about 10μs, would tend to diminish the initial stress jump which was taken as the experimental $\Delta\tau$. All these factors are in the direction of increasing the calculated stress increments relative to the observed ones. The first two are fairly small but the third could be of some significance. These factors might account for most of the differences in results except for the largest ones at 523°K. Further agreement on the stress increments could have been obtained at the expense of poorer matching of the steady rate curves. Reducing the calculated stress increments is achieved mainly by increasing the n values, i.e. lowering the rate sensitivity. A higher n value at 523°K would actually permit better fitting of a hyperbolic relation to the n(T) function, Fig. 9, i.e., eq. (10b) would then have greater validity.

The equations for $Z_i(T)$ and n(T), (9) and (10), obtained from the results of the numerical exercises follow the expected trends that hardness would decrease and rate sensitivity would increase at higher temperatures. That a direct correspondence seems to exist between plastic working and lowering the temperature is an interesting result that could be checked experimentally.

It is difficult to relate eqs. (9) and (10) directly to parameters in the rate equations for plastic flow based on idealized dislocation mechanisms. The exponential form (5) could be approximated by a power law over part of its range where the parameter n would be equal to the power q multiplied by a constant. Campbell shows in [10] that temperature dependence could be approximated in such a power law for constant rate conditions by making $q=q'/kT$. This would correspond to a hyperbolic relation for n(T) such as indicated by (10b). A strain-rate, temperature correspondence in the stress-strain relation is also implied by eqs. (10), but this enters in a more complicated manner than the simple formula proposed in [9].

Results given in [7,8,12] for the metals aluminum, magnesium, zinc, and titanium subjected to rapid changes in strain rate show characteristics that are basically similar to those for copper. Under these circumstances, the constitutive equations described in this paper may be applicable. For certain metals, e.g. mild steel, molybdenum, and niobium, the strain rate jumps sometimes lead to stress levels that exceed the steady rate values [12,13]. These unusual results may be due to strain ageing effects at the lower strain rates which can be incorporated into the constitutive equations.

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List of Captions

TABLE 1

1. Constants in $Z(W_p)$ function for copper (temperature independent):

$$Z = Z_1 + (Z_0 - Z_1)\exp(-mW_p) \tag{6}$$

$$m = m_0 + m_1 \exp(-\alpha W_p) \tag{7}$$

$Z_0 = 31$ (MPa)

$Z_1 = 237$ (MPa)

$m_0 = 1500$ (MPa)$^{-1}$

$m_1 = 2500$ (MPa)$^{-1}$

$\alpha = 5000$ (MPa)$^{-1}$

2. Temperature dependent material constants:

T°K	Z_i (MPa)	n
77	82	14.3
148	49	13.4
298	31	9.2
523	26	3.6

3. Temperature independent material constant:

$D_0^2 = 10^8 \sec^{-2}$

TABLE 2 - COMPARISON OF RESULTS

Temp. (°K)	$\bar{\gamma}_j$(exp)	$\dot{\gamma}_{jd}$(exp) (sec^{-1})	$\Delta\tau$(exp) (MPa)	$\Delta\tau$(calc) (MPa)	$\bar{\tau}_B$(exp) (MPa)	τ_B(calc) (MPa)	$\bar{\tau}_F$(exp) (MPa)	τ_F(calc) (MPa)
77	0.059	100 - 210 157 (avg)	3.44 - 3.62 3.50 (avg)	4.04	71.3	72.5	77.5	77.8
	0.120	130 - 180 155 (avg)	3.44 - 4.20 3.82 (avg)	5.25	97.0	95.1	104.5	102.9
	0.182	60 - 150 105 (avg)	4.13 - 4.48 4.30 (avg)	6.00	119.9	110.7	128.1	118.2
148	0.059	90 - 100 95 (avg)	3.10 - 3.50 3.30 (avg)	3.80	62.1	63.4	68.6	68.6
	0.120	80 - 100 90 (avg)	4.06 - 4.13 4.09 (avg)	5.18	87.2	87.0	97.7	94.9
	0.180	110 - 180 145 (avg)	4.75 - 5.86 5.37 (avg)	6.10	109.4	105.7	122.5	114.0
298	0.061	150 - 180 165 (avg)	2.67 - 3.37 3.02 (avg)	4.63	53.2	52.8	60.5	59.8
	0.120	60 - 400 185 (avg)	3.62 - 5.68 4.61 (avg)	6.53	76.3	75.3	86.0	85.5
	0.182	110 - 150 130 (avg)	5.68 - 6.20 5.94 (avg)	7.89	96.0	94.5	107.9	106.2
523	0.06	150 - 250 200 (avg)	4.48 - 5.24 4.86 (avg)	7.58	31.0	32.6	42.3	45.5
	0.120	60 - 280 170 (avg)	6.27 - 6.37 6.32 (avg)	10.85	48.7	48.3	66.1	66.4
	0.185	110 - 300 205 (avg)	6.41 - 8.27 7.34 (avg)	12.50	63.5	60.9	86.2	83.9

Note: bars indicate average value of several tests; $\bar{\gamma}_j$ = strain at jump; other symbols defined in Fig. 1.

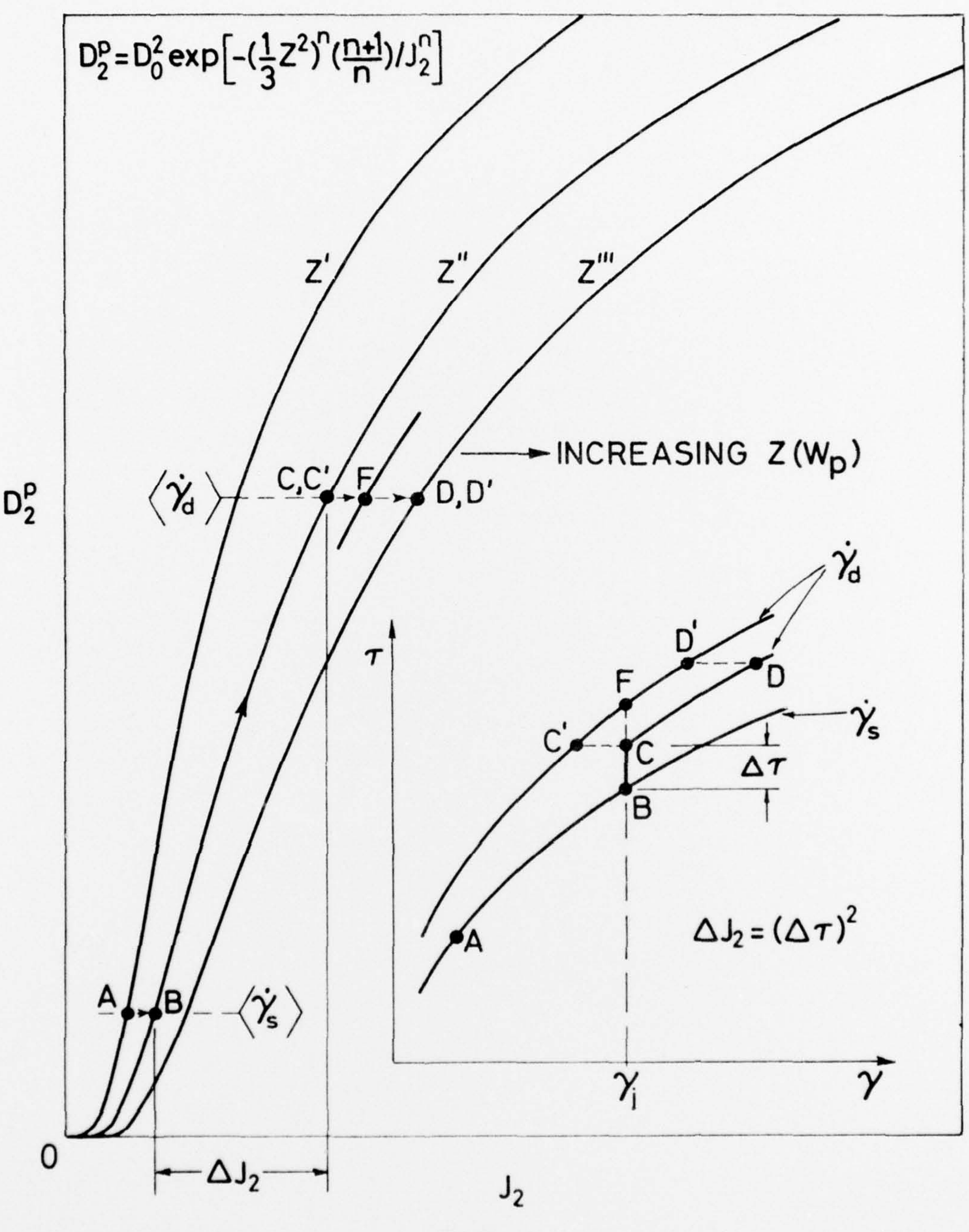

$D_2^p = D_0^2 \exp\left[-(\frac{1}{3}Z^2)^n(\frac{n+1}{n})/J_2^n\right]$
Z'
Z''
Z'''
INCREASING Z(W_p)
D_2^p
$\langle\dot{\gamma}_d\rangle$
C, C'
F
D, D'
A
B
$\langle\dot{\gamma}_s\rangle$
0
ΔJ_2
J_2
τ
$\dot{\gamma}_d$
D'
D
F
C'
C
$\dot{\gamma}_s$
$\Delta\tau$
B
A
$\Delta J_2 = (\Delta\tau)^2$
γ_i
γ

FIG. 1

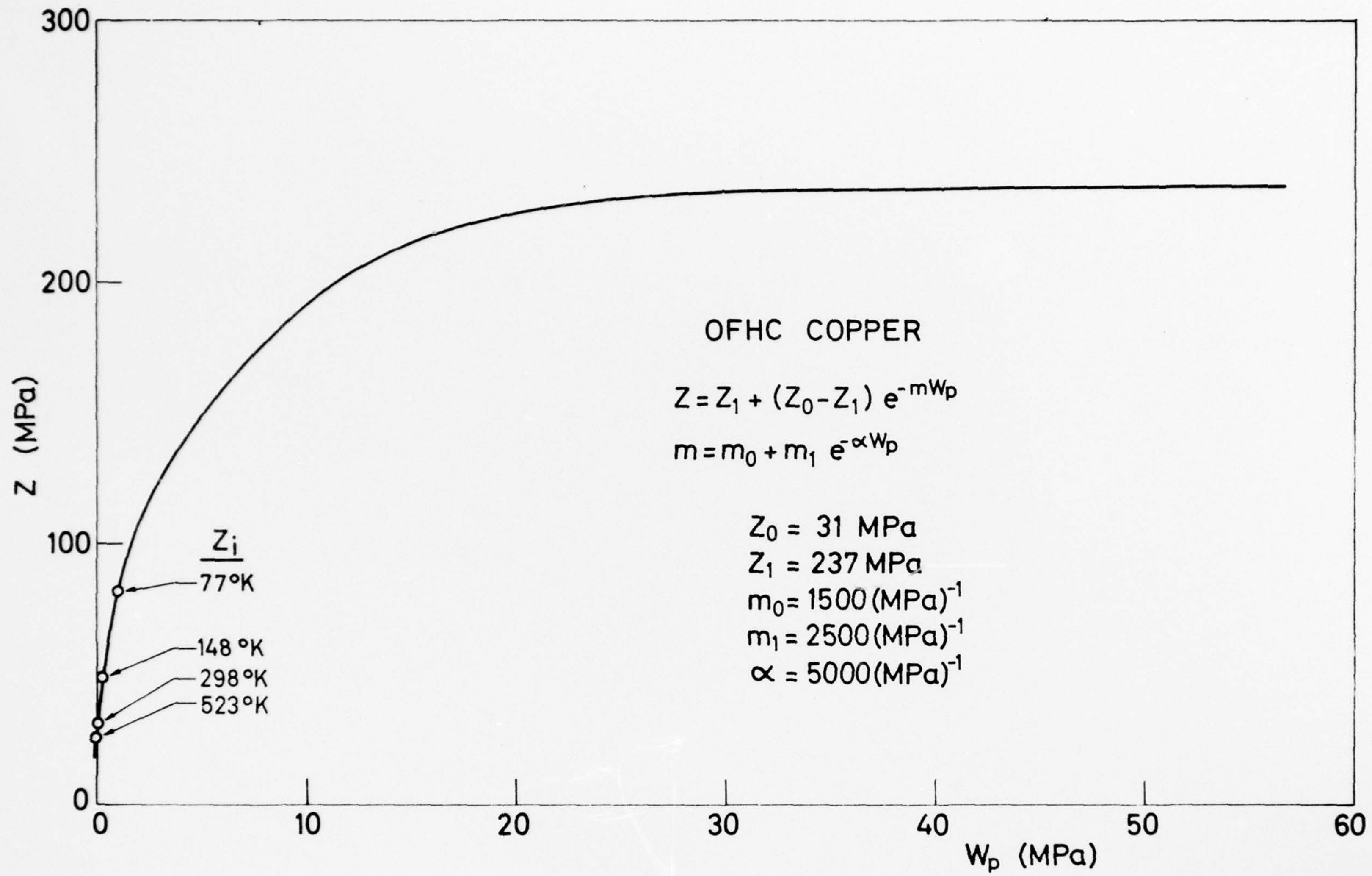

OFHC COPPER
Z = Z1 + (Z0 - Z1) e^-mWp
m = m0 + m1 e^-αWp
Z0 = 31 MPa
Z1 = 237 MPa
m0 = 1500 (MPa)^-1
m1 = 2500 (MPa)^-1
α = 5000 (MPa)^-1
Zi
77°K
148 °K
298 °K
523 °K
Z (MPa)
Wp (MPa)
0
100
200
300
10
20
30
40
50
60

FIG. 2

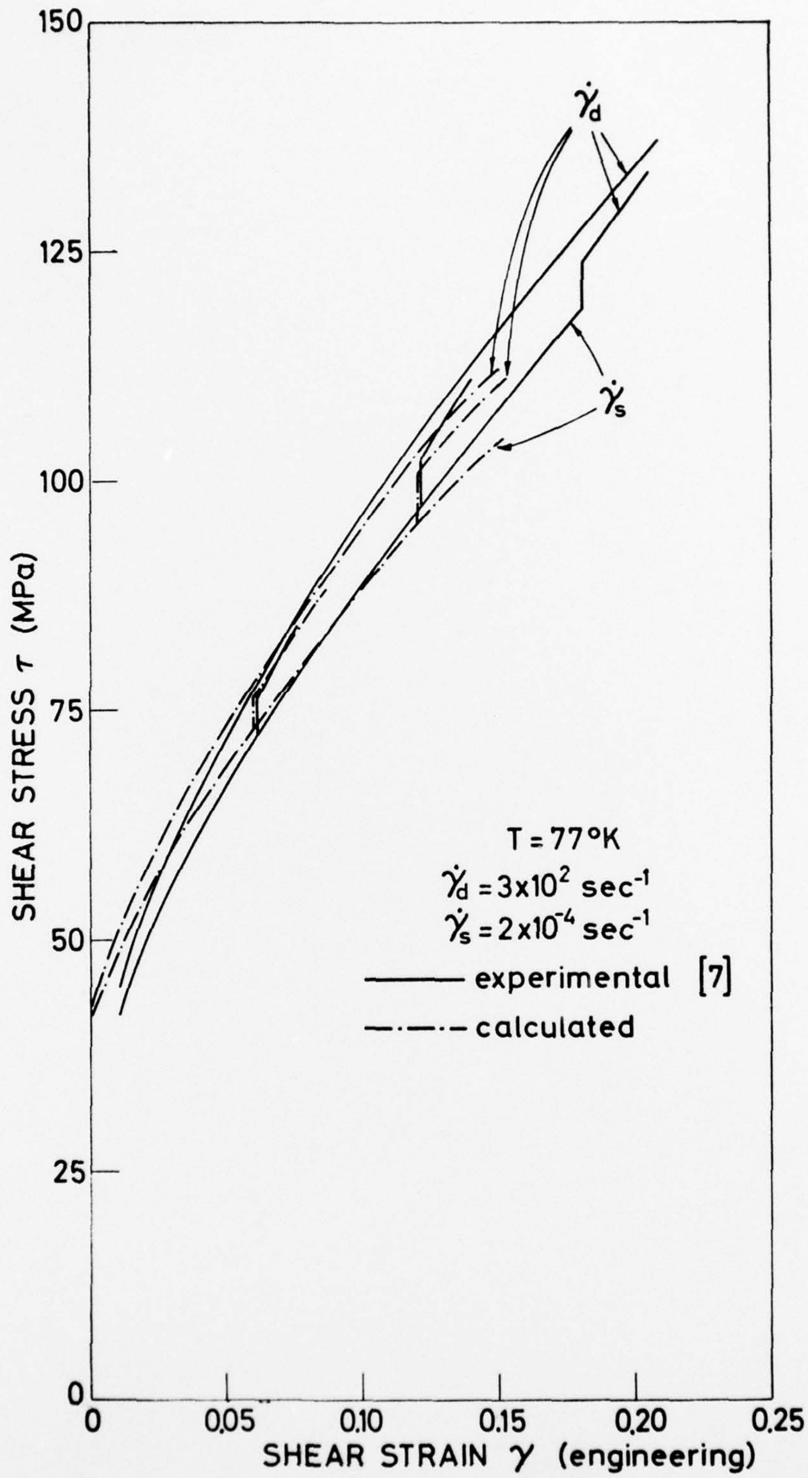
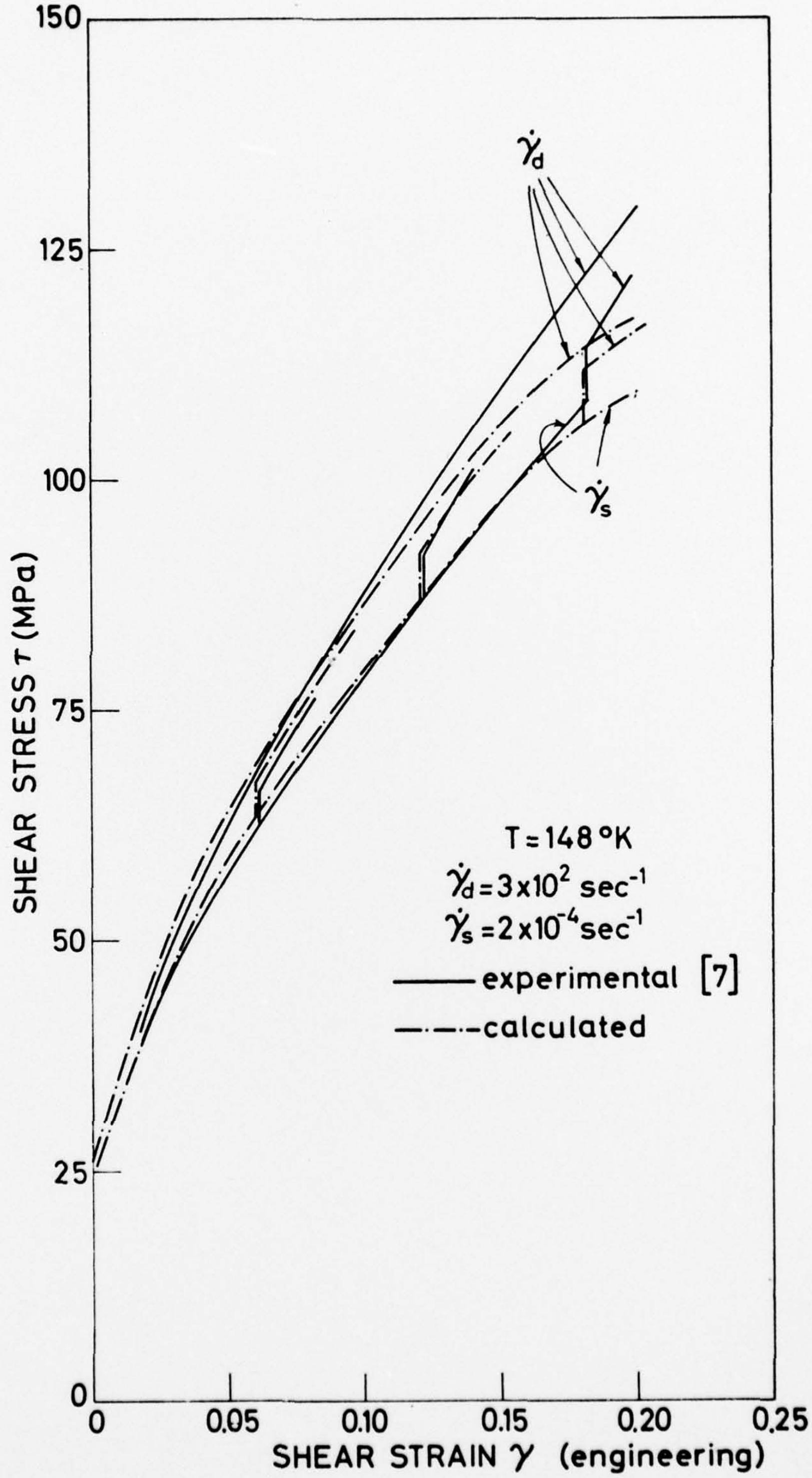
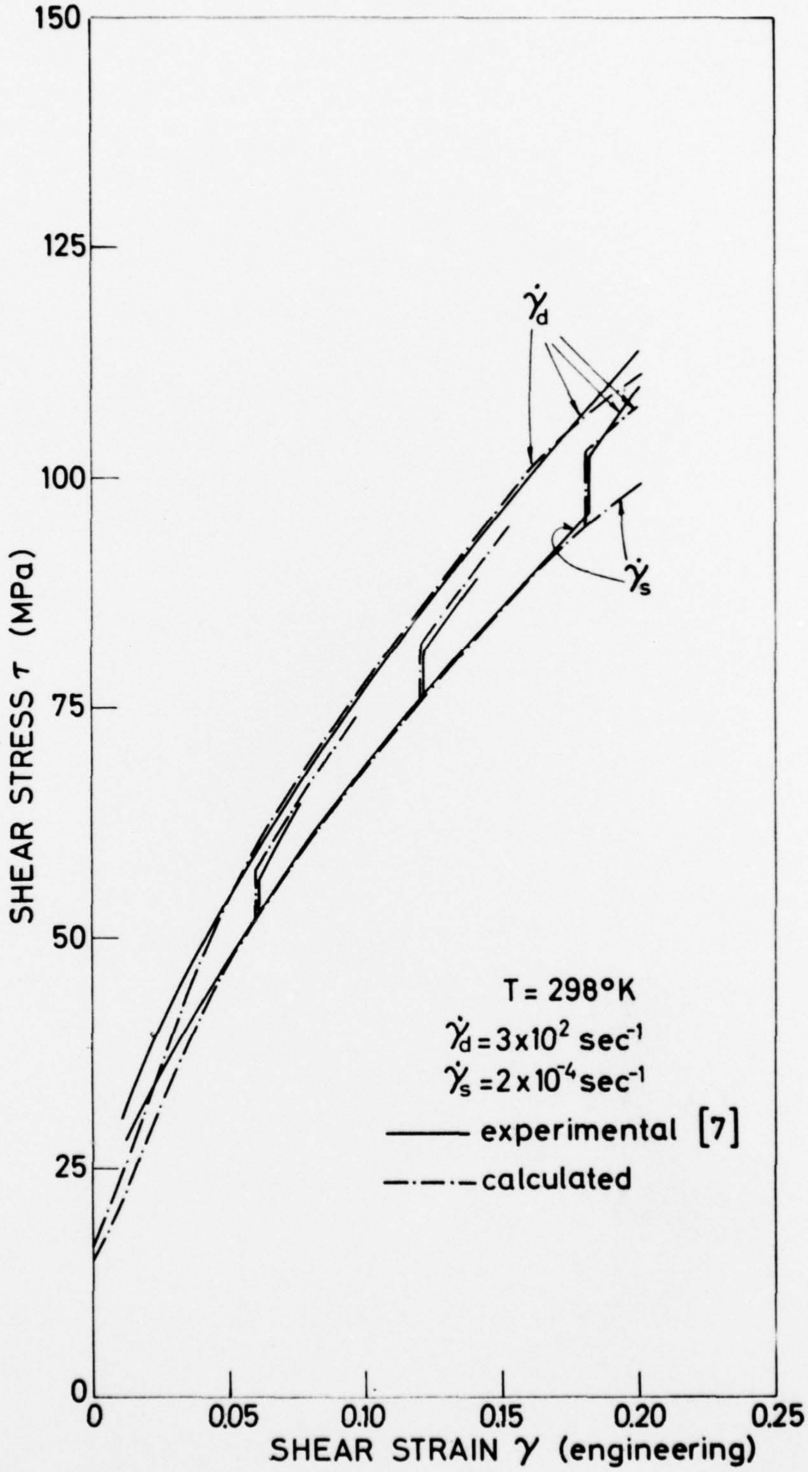

150
125
100
75
50
25
0
0
0.05
0.10
0.15
0.20
0.25
SHEAR STRESS τ (MPa)
SHEAR STRAIN γ (engineering)
$\dot{\gamma}_d$
$\dot{\gamma}_s$
T = 77°K
$\dot{\gamma}_d = 3 \times 10^2$ sec^{-1}
$\dot{\gamma}_s = 2 \times 10^{-4}$ sec^{-1}
experimental [7]
calculated

FIG. 3



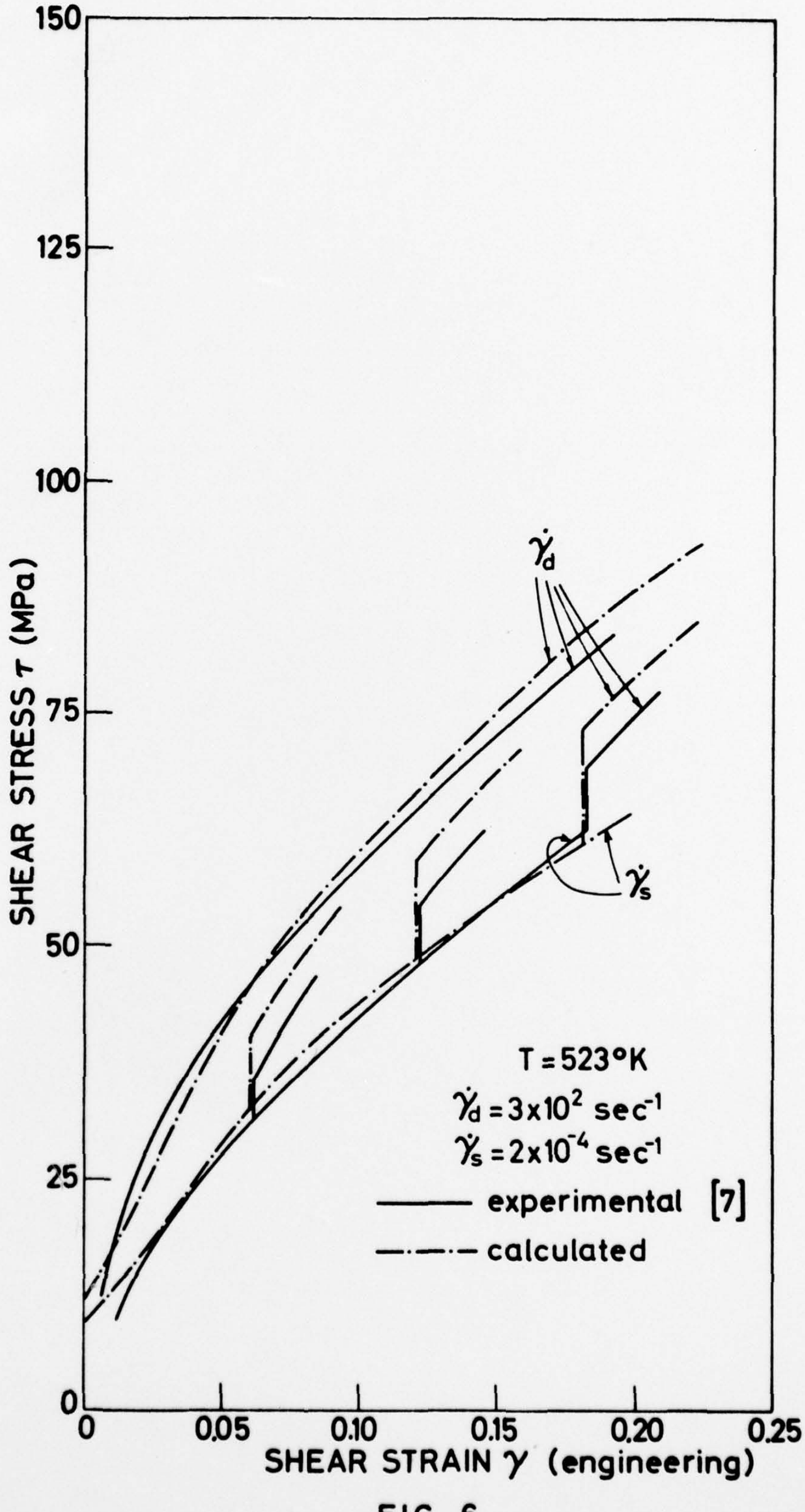
150
125
100
75
50
25
0
SHEAR STRESS τ (MPa)
0
0.05
0.10
0.15
0.20
0.25
SHEAR STRAIN γ (engineering)
$\dot{\gamma}_d$
$\dot{\gamma}_s$
T = 148 °K
$\dot{\gamma}_d = 3 \times 10^2\ sec^{-1}$
$\dot{\gamma}_s = 2 \times 10^{-4} sec^{-1}$
experimental [7]
calculated

FIG. 4



150
125
100
75
50
25
0
0
0.05
0.10
0.15
0.20
0.25
SHEAR STRESS τ (MPa)
SHEAR STRAIN γ (engineering)
$\dot{\gamma}_d$
$\dot{\gamma}_s$
T = 298°K
$\dot{\gamma}_d = 3 \times 10^2$ sec^{-1}
$\dot{\gamma}_s = 2 \times 10^{-4}$ sec^{-1}
experimental [7]
calculated

FIG. 5



150
125
100
75
50
25
0
SHEAR STRESS τ (MPa)
0
0.05
0.10
0.15
0.20
0.25
SHEAR STRAIN γ (engineering)
$\dot{\gamma}_d$
$\dot{\gamma}_s$
T = 523°K
$\dot{\gamma}_d = 3 \times 10^2$ sec^{-1}
$\dot{\gamma}_s = 2 \times 10^{-4}$ sec^{-1}
experimental [7]
calculated

FIG. 6

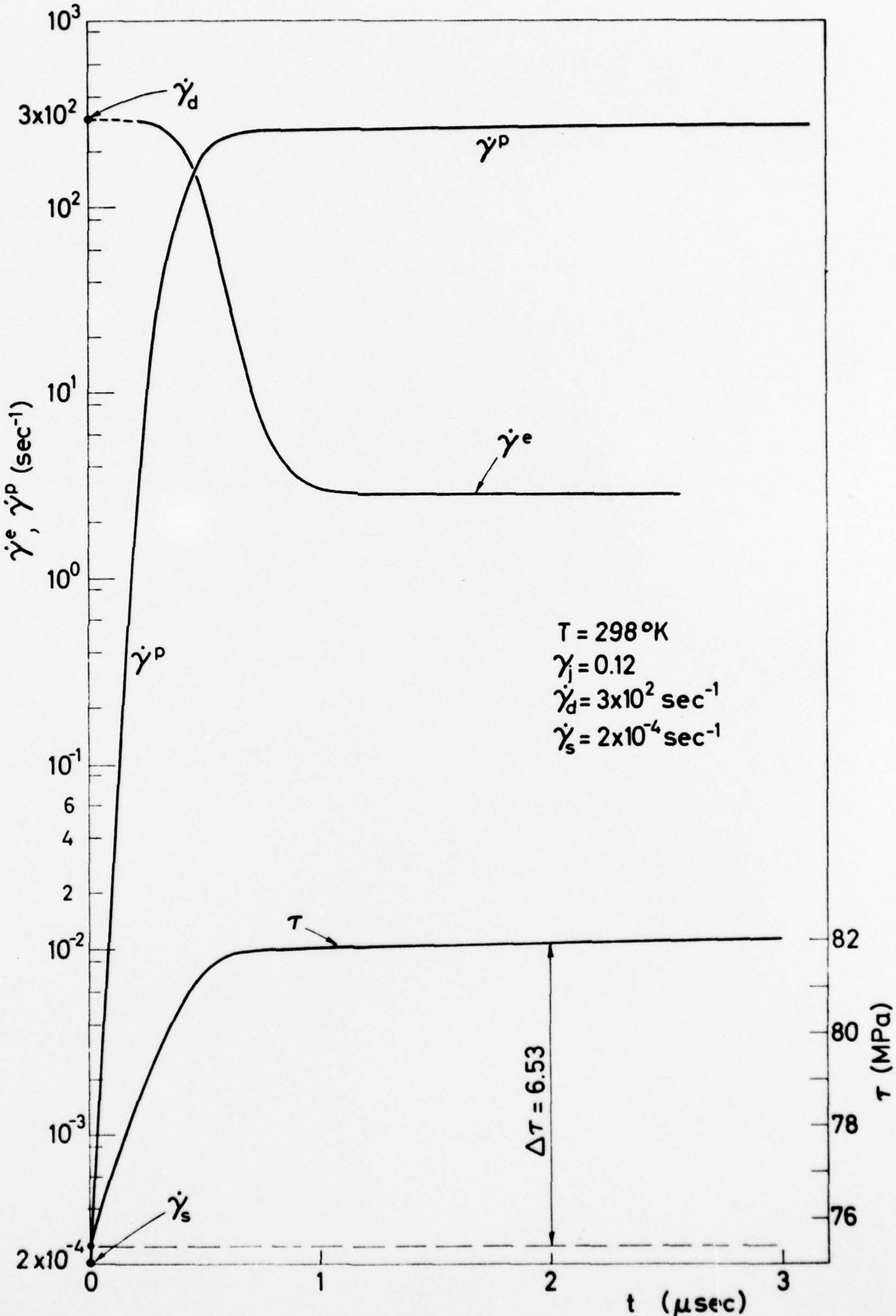

10^3
$\dot{\gamma}_d$
3×10^2
$\dot{\gamma}^p$
10^2
10^1
$\dot{\gamma}^e$
$\dot{\gamma}^e, \dot{\gamma}^p\ (\mathrm{sec}^{-1})$
10^0
$T = 298\,^\circ K$
$\gamma_j = 0.12$
$\dot{\gamma}_d = 3\times10^2\ \mathrm{sec}^{-1}$
$\dot{\gamma}_s = 2\times10^{-4}\ \mathrm{sec}^{-1}$
10^{-1}
6
4
2
τ
10^{-2}
82
80
78
76
τ (MPa)
$\Delta\tau = 6.53$
10^{-3}
$\dot{\gamma}_s$
2×10^{-4}
0
1
2
3
t (μsec)

FIG. 7

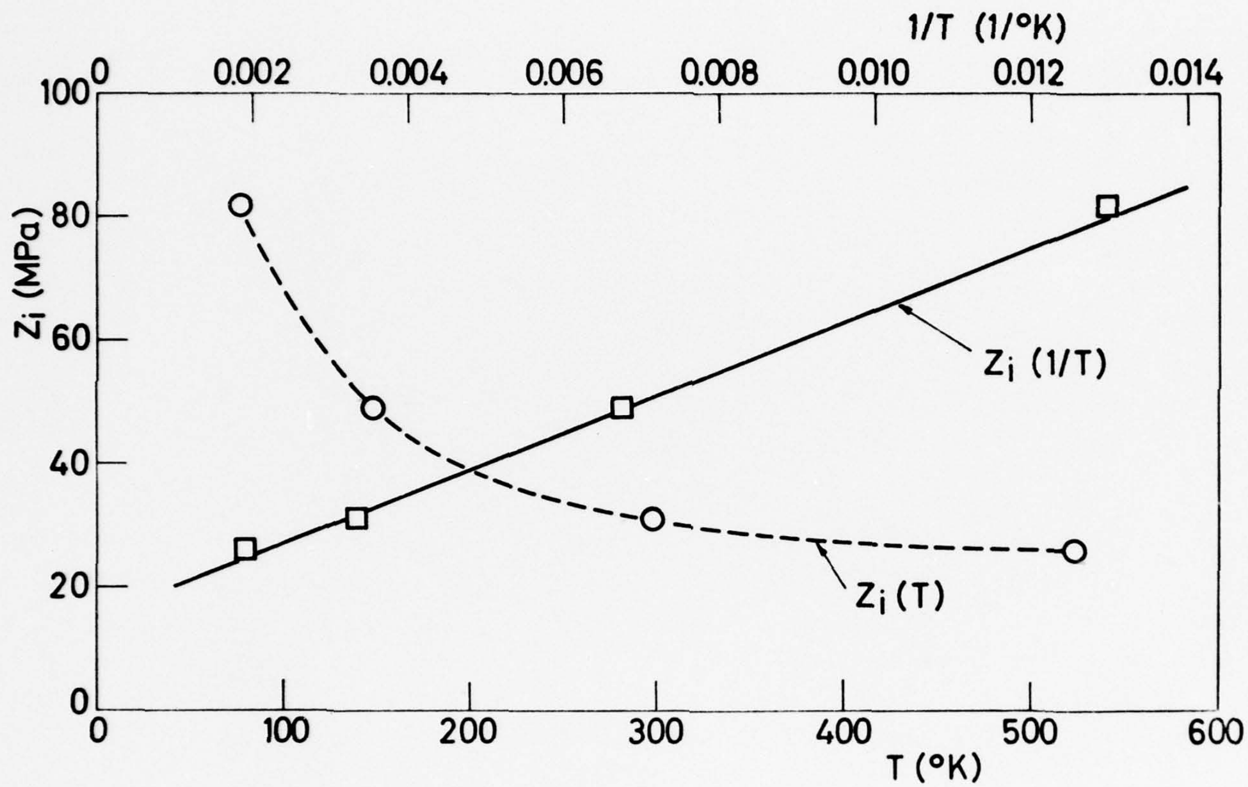

1/T (1/°K)
0
0.002
0.004
0.006
0.008
0.010
0.012
0.014
100
80
60
40
20
0
Zi (MPa)
Zi (1/T)
Zi (T)
0
100
200
300
400
500
600
T (°K)

FIG. 8

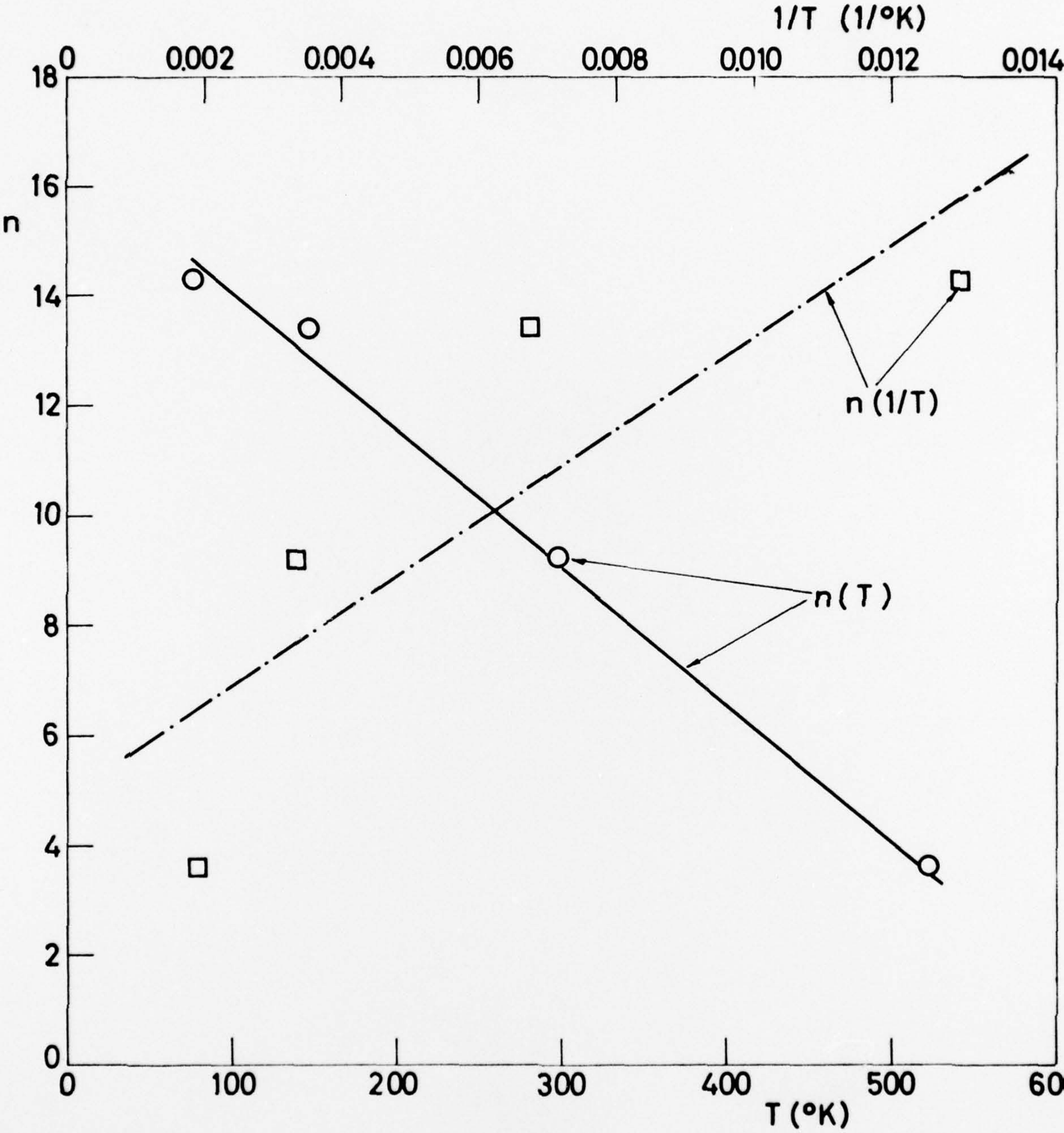

1/T (1/°K)
0
0.002
0.004
0.006
0.008
0.010
0.012
0.014
n
18
16
14
12
10
8
6
4
2
0
n (1/T)
n (T)
0
100
200
300
400
500
600
T (°K)

FIG. 9

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18. SUPPLEMENTARY NOTES

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VISCOPLASTICITY
CONSTITUTIVE EQUATIONS
COPPER
STRAIN RATE EFFECTS
STRAIN RATE HISTORY
TEMPERATURE EFFECTS

* .0002/sec to 300/sec

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Elastic-viscoplastic constitutive equations based on a single internal state variable which is a function of plastic work are used to calculate the response of copper to a six decade change of strain rates over a range of temperatures. Calculations were performed for the conditions of an experimental program on copper by Senseny, Duffy, and Hawley, namely, temperatures ranging from 77 to 523°K and strain rate jumps from ~~$2x10^{-4}sec^{-1}$ to $3x10^{2}sec^{-1}$~~ at three strain levels. The computed results are in good agreement with the experiments and show similar strain rate and strain rate history effects. Relations are obtained for the temperature dependence of certain parameters in the equations which

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indicate correspondence between plastic working and temperature and between strain rate sensitivity and temperature.